Biologic - An Introduction

Louis H. Kauffman
Department of Mathematics, Statistics
and Computer Science (m/c 249)
851 South Morgan Street
University of Illinois at Chicago
Chicago, Illinois 60607-7045
<kauffman@uic.edu>

January 30, 2016

Abstract

We explore the boundary shared by biology and formal systems.

Keywords. logic, algebra, topology, biology, replication, celluar automaton, quantum, DNA, container, extainer

1 Introduction

This essay is an introduction to my research on the mathematics of self-reference, self-replication and its applications to molecular biology. This introduction is based on my paper [22] and the reader is encouraged to examine that paper. Other relevant papers will be found in the bibliography of this paper. We mention particularly [24, 23, 26, 25, 21, 17, 18].

I will concentratie here on relationships of formal systems with biology. In particular, this is a study of different forms and formalisms for replication. See previous papers by the author [25, 24, 23]. We concentrate here on formal systems not only for the sake of showing how there is a fundamental mathematical structure to biology, but also to consider and reconsider philosophical and phenomenological points of view in relation to natural science and mathematics. The relationship with phenomenology [37, 35, 36, 9, 1, 39] comes about in the questions that arise about the nature of the observer in relation to the observed that arise in philosophy, but also in science in the very act of determining the context and models upon which it shall be based. Our original point of departure was cybernetic epistemology [44, 43, 41, 34, 12, 13, 14, 15, 16, 17, 18, 24, 23, 25] and it turns out that cybernetic epistemology has much to say about the relation of the self to structures that may harbor a self. It has much to say about the interlacement of selves and organisms. This study can be regarded as an initial exploration of this theme of mathematics, formalities, selves and organisms - presented primarily from the point of view of cybernetic epistemology, but with the intent that these points of view should be of interest to phenomenologists. We hope to generate fruitful interdisciplinary discussion in this way.

Our point of view is structural. It is not intended to be reductionistic. There is a distinct difference between building up structures in terms of principles and imagining that models of the world are constructed from some sort of building-bricks. The author wishes to make this point as early as possible because in mathamatics one naturally generates hierarchies, but that does not make the mathematician a reductionist. We think of geometry as the consequences of certain axioms for the purpose of organizing our knowledge, not to insist that these axioms are in any way other than logically prior to the theorems of the system. Just so, we look for fundamental patterns from which certain complexes of phenomena and ideas can be organized. This does not entail any assumption about "the world" or how the world may be built from parts. Such assumptions are, for this author, useful only as partial forms of explanation.

We examine the schema behind the reproduction of DNA. As all observers of science know, the pattern of the DNA reproduction is very simple. The DNA molecule consists of two interwound strands, the Watson Strand (W) and the Crick Strand (C). The two strands are bonded to each other via a backbone of base-pairings and these bonds can be broken by certain enzymes present in the cell. In reproduction of DNA the bonds between the two strands are broken and the two strands then acquire the needed complementary base molecules from the cellular environment to reconstitute each a separate copy of the DNA. At this level the situation can be described by a symbolism like this.

$$DNA = < W|C > \longrightarrow < W| \ E \ |C > \longrightarrow < W|C > < W|C > = DNA \ DNA.$$

Here E stands for the environment of the cell. The first arrow denotes the separation of the DNA into the two strands. The second arrow denotes the action between the bare strands and the environment that leads to the production of the two DNA molecules.

Much is left out of this schema, not the least of which is the ignoring of the word *interwound*. Indeed the DNA molecule is a tight spiral winding of its two interlocked strands and so the new DNA's would be linked around one another if it were not for the work of other enzymes that mysteriously manage to unlink the new DNA's in time for cell division to occur. Nevertheless, this is the large scale description of the replication of DNA that is fundamental to the division of cells and to the continuance of living organisms.

The abstract structure of this DNA replication schema makes it a pivot to other models and other patterns. To see this most clearly, suppose we have O and O^* algebraic entities such that $O^*O = 1$ where 1 denotes an algebraic identity such that $1A = A1 = A$ for any other algebraic entity A. Assume that juxataposition (multiplication) of algebra elements is associative. Let

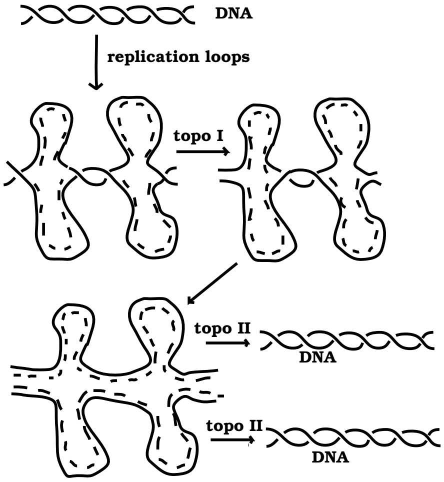

Figure 1: **DNA Replication**

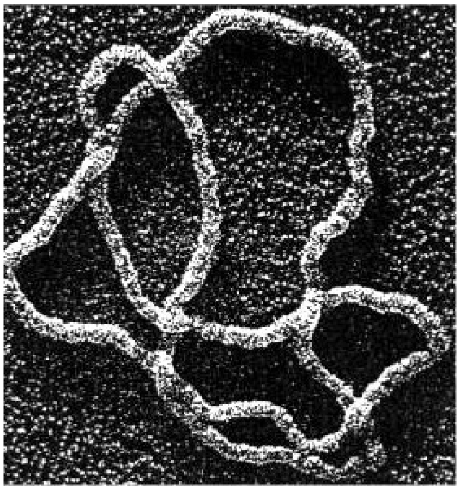

Figure 2: **DNA Knot**

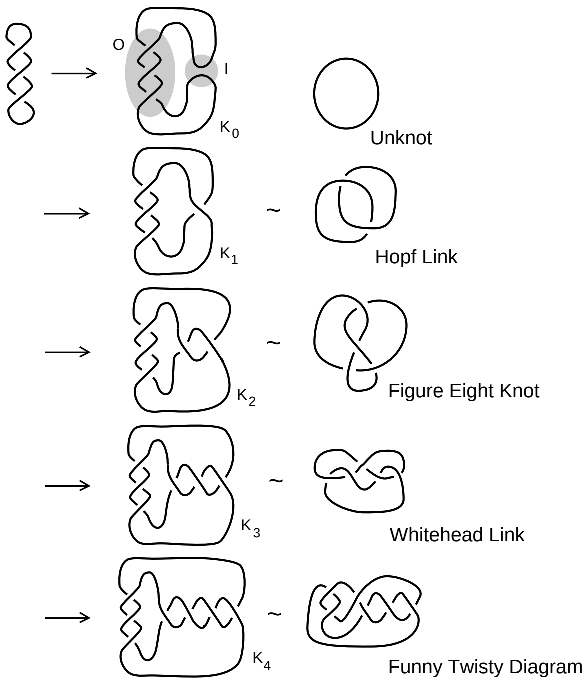

Figure 3: **Recombination Process**

$P = OO^*$. Then

$$P = OO^* = O1O^* = OO^*OO^* = PP.$$

Thus we have a general algebraic form for the self-replication described above. Note that in algebra we do not choose a direction of change. Thus we have $1 = O^*O$. In the replication scenario this is replaced by a *process arrow*

$$1 \longrightarrow O^*O$$

generalizing the arrow

$$E \longrightarrow |C >< W|$$

where the environment E can supply Crick and Watson strands (via the base pairing) to the opened DNA. Thus algebra provides a condensed formalism for discussing self-replication.

We now invite the reader to examine the form of the science involved in this well-known description. We speak of the DNA molecules as though we could see them directly in the phenomenology of our ordinary sight. Some science does involve the direct extension of sight as the experience of looking through a telescope or a light microscope. But in the case of the DNA one proceeds by logical consistency and the indirect but vivid images via the electron microscope and the patterns of gel electrophoresis. In the case of electron microscope images there is every reason to assume (that is, it appears consistent to assume) that the objects shown can be taken to be analogous to the macroscopic objects of our perception. This means that one has the possibility of observing "directly" that DNA molecules can be knotted. I do not say that one can observe directly the coiling of the Watson and the Crick strands, but the full DNA can be observed as though it were a long rope. This rope can be seen to be coiled and knotted in electron micrographs such as the one shown here in Figure 2. Even this "showing" requires a difficult technique beyond the usual techniques of the electron microscope. The DNA was coated with protein by the experimenters so that it became a chain of larger and more robust diameter. Then the electron microscope revealed the patterns of knotting in an apparent projection of the coated DNA from three dimensional space to the two dimensional space of the image.

Along with these forays into experimentation, there are also analogous forays into the limits of logic. Here we meet the replication schema again. Replication in logic is intimately related to self-reference and to formalisms that, if not properly interpreted, can lead to paradox. The reasons for this are, by now, apparent. The usual mathematical formalisms for set theory assume that there is no temporal evolution in the structures. The sets do not change over time. A set like the Russell *set of all sets that are not members of themselves* crosses the boundary of such restrictions. Once the Russell set is declared, the set itself comes under scrutiny for the very property that defines it. In this case, if we think recursively, the new Russell set is not a member of itself, but it is a new set, just created. And so we must take a step, and form a new Russell set that includes the first one. This new Russell set is also subject to scrutiny and must be further included in a yet again new Russell set. The process continues ad infinitum. A declaration of set membership has led to a recursive process of self-production. This may look like a tragedy for the classical mathematics, but it is exactly what interests us when studying biology! Mathematical Biology is concerned with those abstract structures leading to recursive generation of structures from themselves and from their environments. For this reason we explore such abstract schema in this paper.

A simplest form of recursive replication is formalized by the following consideration: Suppose that we have a domain D (a *reflexive domain*) where every element $a \in D$ is also seen as a mapping from D to D,

$$a : D \longrightarrow D.$$

We let $[D, D]$ denote the collection of all mappings from D to itself. In a reflexive domain, we have an identification of the domain with the transfomations of that domain:

$$D = [D, D].$$

For example, suppose that a is any element of D. Define a new element $G \in D$ by the equation

$$Gx = a(xx).$$

(Our reflexive assumption guarantees the existence of such a G.) Then

$$GG = a(GG).$$

One can read this equation as saying that *every $a \in D$ has a fixed point of the form GG for an element $G \in D$.* The element GG is *productive* in that it produces an a and will continue to do so.

$$GG \longrightarrow a(GG) \longrightarrow a(a(GG)) \longrightarrow a(a(a(GG))) \longrightarrow \cdots.$$

Here the arrow is intended to indicate the process of production of the $a.$ GG is like a cell that can divide, and once it divides it can divide again. It is not lost upon us that GG as an abstract cell is its own genetic material and *doubled* (two G's) in an abstract hint of the double helix of $DNA.$

The mathematical phenomenology of this fixed point construction can be illustrated by a shift of notation. Define

$$Gx = \langle xx \rangle.$$

As long as x is not $G,$ then this operator seems quite innocuous, but when we allow $x = G$ then we have

$$GG = \langle GG \rangle,$$

and the form GG has miraculously appeared inside itself. The notational shift is effective when the reader takes on the brackets as an enclosure, for then he can be surprised that a form would enclose itself. I intend to give the reader a phenomenological shock of this kind by using the shift of notation. The shift is not necessary for the shock, but anyone who sees the Church-Curry fixed point construction and is not shocked, has not seen the story to its roots. Here is the problem of understanding laid bare. We can tell a joke, but will the listener get the joke? What does it mean to get the joke?

I say that GG is an *eigenform* for the operator $T(x) = \langle x \rangle$ since it is a fixed point for that operator. See [12, 13, 14, 15, 16, 17, 18] and note that a fixed point V such that $T(V) = V$ is analogous to an eigenvector with eigenvalue equal to one. Thus one can think of the eigenforms associated with a given transformation as correpondent to a generalized *spectrum* of the operator $T.$ Eigenforms go beyond numerical spectra to Fixed Points in larger domains. It is still spectral analysis of a kind, but Eigenforms speak to the arising of 'objects as tokens for eigenbehaviours' where an eigenbehaviour is a behaviour that has the character of a fixed point even when that fixed point is in a newly created domain not part of the status quo of the transformations that engendered it. There are many examples. For example, Heinz von Forerster points out that the sentence "I am the observed relation between myself and observing myself." defines the concept of "I" as an eigenform of the transformation

$T(x) =$ "The observed relation between x and observing x." [44].

In the arising of a solution to the equation

$$I = T(I),$$

an “I” comes into being. These ‘I’ are not part of the status quo of the systems that engender them. They are transcendent to those systems, and are often seen as illusory or otherwise magical. But one can also regard the ‘I’ as the direct result of the action of the organism itself. Thus here we find a nexus that allows many points of view, from the classical transcendental view of the self to the intertwined phenomenological view of self and world in mutual embrace and mutual creation as in the work of Merleau-Ponty [35]. Observing systems can have “I”s but they do not produce them. They are them. This says a great deal about the efficacy of using cybernetic epistemology to understand understanding. We hope that the reader will bear with these attempts at comparison that will surely become more systematic in later work. These comparisons are important particularly in facing the question of how organisms acquire awareness and how awareness can be applied to itself.

Remark. In a purely formal treatment or in a computer program, one must take care of the possibility of uncontrolled recursion. It is worth noting that at the linguistic level, there is no intent to repeat in “I am the one who says I.”. Another example of this “stopping” is the famous sentence due to Quine:

Refers to itself when appended by its own quotation “refers to itself when appended to its own quotation.”

There is no necessity for an uncontrolled recursion to occur at the point of self-reference or self-replication. It is a matter of context. In the case of DNA reproduction the replication happens in the cell only under very special conditions, and it is immediately followed by the separation of the new DNA’s into their respective new cells. The new cells can then undergo mitosis again, but that self-replication is dependent upon the possibilities in the environment.

Theoretically, uncontrolled recursion leads to the notion of fixed points in a direct manner by talking the limit of iterated recursion. Consider the transformation

$$F(X) = \left.\overline{\;\mathrm{X}\;}\right|.$$

If we iterate it and take the limit we find

$$G = F(F(F(F(\cdots)))) = \left.\overline{\left.\overline{\left.\overline{\left.\overline{\;\cdots\;}\right|}\right|}\right|}\right|$$

an infinite nest of marks satisfying the equation

$$G = \left.\overline{\;\mathrm{G}\;}\right|.$$

With $G = F(G)$, I say that G is an *eigenform* for the transformation F, and this is an eigenform that occurs by taking a limit of the recursion. See Figure 4 for an illustration of this nesting with boxes and an arrow that points inside the reentering mark to indicate its appearance inside itself. If one thinks of the mark itself as a Boolean logical value, then extending the language to include the reentering mark G goes beyond the boolean. We will not detail here how this extension can be related to non-standard logics, but refer the reader to [25].

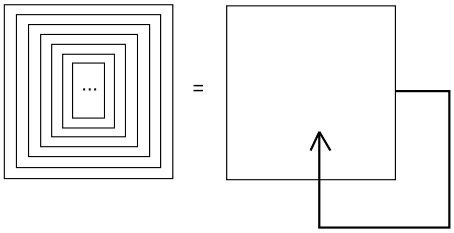

Figure 4: **An Infinite Fixed Point**

The interesting thing about such recursive processes is that in the abstract they do not stop. The abstraction that never stops is behind the actual processes that stop eventually due to lack of resources or other means. Just as one may consider such abstractions, one can idealize process by taking it to infinity. In the case of GG we see that we can consider a formal fixed point of a in the form

$$P = a(a(a(a(a(a(a(a(a(a(a(a(a(a(\cdots)))))))))))))).$$

In a microcosm, such patterns are the fractal residue of recursive processes in organisms. They do not reach ideal infinity in actuality, but the ideal lies behind the real and has its own mathematical reality. The reader can see how, in the cybernetic epistemology, the statement that the ideal lies behind the real takes the form of the interlacement of recursion and the generation of the object as a token for the behavior of that recursion. Without an interaction of subject and object neither subject nor object can come forth into a world, nor can that world come forth. This is not to say that there is no background to these circularities. It is a question of attending to process and mutuality to realize the creative nature of the world into which the subject is thrown and the nature of the subject into which the world is thrown.

Interpretation of this basic semantic/syntactic level yields untold riches. For example, consider the von Neumann schema for a Universal Building Machine denoted B. B will produce any entity X for which there is a blueprint x. We write

$$B, x \longrightarrow X, x.$$

The machine B produces the entity X with a copy of its blueprint x attached. Now let b be the blueprint for B itself. Then we have

$$B, b \longrightarrow B, b.$$

The machine B has reproduced itself. The *pattern* of this reproduction is the same as the pattern for the reflexive fixed point. We might have written

$$Bx \longrightarrow xx$$

and

$$BB \longrightarrow BB.$$

The separation of entity and blueprint is our distinction. In biology the two strands of DNA are each blueprint and entity. Nature begins without the logical distinctions that we find so compelling.

Reflexive domains are daring in their nature. They dare to allow process and form to coexist. They dare to combine time and timelessness. To see the fine wire we are wallking, the reader should consider a reflexive domain that allows the operation of negation $\sim$. Such a domain could be one that allows logical discourse among its operators. We then define

$$Rx = \sim xx,$$

and find that

$$RR = \sim RR.$$

Thus we find an element RR in this logical reflexive domain that is its own negation. Such objects are not allowed in classical logical domains. To see this circularity to its bitter end, interpret Ax as "x is a member of A.". Then R is identified as the "set of all x that are not members of themselves", and we see that we have reproduced the Russell Paradox. For us the way out is via the recursion. But this requires further discussion for which biology and its lessons can help.

In living systems there is an essential circularity that is the living structure. Living systems produce themselves from themselves and the materials and energy of the environment. There is a strong contrast in how we avoid circularity in mathematics and how nature revels in biological circularity. One meeting point of biology and mathematics is knot theory and topology. This is no accident, since topology is indeed a controlled study of cycles and circularities in primarily geometrical systems.

In the end we arrive at a summary formalism, a chapter in *boundary mathematics* (mathematics using directly the concept and notation of containers and delimiters of forms - compare [4] and [41]) where there are not only containers $<>$, but also extainers $><$, entities open to interaction and distinguishing the space that they are not. In this formalism we find a key for the articulation of diverse relationships. The *boundary algebra of containers and extainers* is to biologic what boolean algebra is to classical logic. Let $C \; = <>$ and $E \; = ><$. Then

$$EE \; = ><>< = > C <$$

and

$$CC \; = <><> = < E > .$$

Thus an extainer produces a container when it interacts with itself, and a container produces an extainer when it interacts with itself. The formalism of containers and extainers can be compared with Heidegger's lifeworld of objects sustaining each other through mutual transpermeation [37], a mutual interpenetration that gives rise to form.

The formalism of containers and extainers is a chapter in the foundations of a symbolic language for shape and interaction. With it, we can express the *form* of DNA replication succinctly as follows: Let the DNA itself be represented as a container

$$\text{DNA } = <> .$$

We regard the two brackets of the container as representatives for the two matched DNA strands. We let the extainer $E = ><$ represent the cellular environment with its supply of available base pairs (here symbolized by the individual left and right brackets). When the DNA strands separate, they encounter the matching bases from the environment and become two DNA's.

$$\text{DNA} = <> \longrightarrow < E > \longrightarrow <><> = \text{ DNA DNA.}$$

Life itself is about systems that search and learn and become. The little symbol

$$E = ><$$

with the property that

$$EE = ><><$$

producing containers $<>$ and retaining its own integrity in conjunction with the autonomy of $<>$ (the DNA) can be a step toward bringing formalism to life.

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