BIOLOGIC-Replication and Self-Reference in Formal Systems and Biology Louis H. Kaufman, UIC kauffman@uic.edu loukau@gmail.com www.math.uic.edu/ ~kauffman

Classical Aspects:

Self-Reference, Recursion, Imaginary Values.

Symbols and reproducibility of symbols.

Separation of object and reference. A circle (intended to refer to distinction) can be regarded as referring to itself as a distinction.

Church-Curry Fixed Point Theorem.

In a reflexive domain any element has a fixed point.

The von-Neumann Building Machine can build itself.

 B ,x $\longrightarrow B$,x X ,x

 $(x$ is the blueprint for X) Let b be the blueprint for B. Then B,b builds itself.

 $B,b \longrightarrow B,b \ B,b$

Note that the incompleteness phenomenon does not occur at level of indicative shift. It occurs at the cut between formal system and observer of formal system.

A universal observer UO examines what is in his hand.

F

He sees a formal system F and wonders if perhaps F is the complete model of his ability to reason.

UO

But no, it cannot be! For UO can prove the incompleteness of F and so UO knows that he is not identical with F, just so long as UO and F are both consistent in their reason.

F

The UOF is not a UFO, but she is beyond Turing. Is the necessary cognition for this ability related to biology and/or (Penrose) new physics?

UO

Self Reference occurs at the Shift of the Name M of the Meta-Naming Operator #.

> " I am the Observed relation Between myself And Observing myself." (Heinz von Foerster)

So far, this is the story of the classical logic of self-replication and self-reference.

We know that DNA engages in self-replication.

How does the DNA self-rep compare with our familiar self-replication at the level of logic and recursion?

$\mathsf{D}\mathsf{N}\mathsf{A}$ is a Salf-Raplicating Form the other the model is the complete with strand. Absorption we can write σ larger pieces of separation as well. Once the strands are separated, the environment of Doplicating Form bases to form the bases of the base pairs $\mathsf{Liplicating}$ romants in virtual, finds its complement being its complement DNA is a Self-Replicating Form

the state of t

 $DNA=$ When $\mathcal{L}_\mathcal{A}$ is the set of the set of

built naturally in the environment. This picture ignores the well-known topological

DNA consists in two strands of base-pairs wound helically around a phosphate

The base pairs are AT (Adenine and Δ Guanine and Δ Guanine and Δ

to symbolize the binding of the two strands into the single DNA duplex. Replication occurs via the separation of the two strands via polymerase enzyme. This separation occurs locally and propagates. Local sectors of separation can amalgamate into

of the cell can provide each with complementary bases to form the bases to form the bases to form the bases pairs of α new duplex DNA's. Each strand, separated in vivo, finds its complement being built naturally in the environment. This picture ignores the well-known topological

|C >= |...AATCTTATCCATGCGC... > . $|C\rangle = |...AATCTTATCCATGCGC...$

 $\langle W| + E \longrightarrow \langle W|C \rangle = DNA$

 $\langle E, W | C \rangle \longrightarrow \langle W | + E + | C \rangle \longrightarrow \langle W | C \rangle \langle W | C \rangle$

∞ can let ∞ and ∞ stall the complete the complete pairs and Critis Ag, TC to the Watson and Crick strands. The Watson School is t
Strands. And Critis Strands. And Critis strands. The Crick strands. The Crick strands. The Crick Strands. The $\sum_{n=1}^{\infty}$ \mathcal{S}_{max} Self Replication Schematic

 $W_{\rm eff}$, which is the set of $W_{\rm eff}$ and $W_{\rm eff}$ and $W_{\rm eff}$ are $W_{\rm eff}$ and $W_{\rm eff}$

DNA = <Watson|Crick> $E =$ Environment $\mathbf{x} = \mathbf{x} - \mathbf{x}$ as $\mathbf{x} = \mathbf{x} - \mathbf{x}$ as $\mathbf{x} = \mathbf{x} - \mathbf{x}$ as $\mathbf{x} = \mathbf{x} - \mathbf{x}$ $\sum_{i=1}^{n}$ are AT (Adenoising are AT (Guanine and $\sum_{i=1}^{n}$

The base pairs are AT (Adenine and Thymine) and GC (Guanine and Cytosine). Thus if where $\frac{w}{\tan \theta}$ and $\frac{w}{\tan \theta}$ and $\frac{w}{\tan \theta}$ and $\frac{w}{\tan \theta}$ and $\frac{w}{\tan \theta}$

 $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$

which the environment supplies the environmental supplies the complementary base pairs AG, TC to the complemen
TC to the complementary base pairs AG, TC to the environmental supplies that the environmental supplies that t

Watson and Crick strands. In this oversimplification we have cartooned the

Compare this method of replication with the movements of the universal

building machine supplied with its own blue product in the supplied with its own blue product in the supplied w
Here Watson and Crick with its own blue product in the supplied with its own blue product in the supplied with

$$

$$

Then

 $|C> = |...AATCTTATCCATGCC...>.$

Symbolically we can oversimplify the whole process as \hfill

 $\langle W| + E \longrightarrow \langle W|C \rangle = DNA$ $E + |C> \longrightarrow = DNA$ $\langle W|C\rangle \longrightarrow \langle W| + E + |C\rangle \rangle = \langle W|C\rangle \langle W|C\rangle$ $\leq W + E \longrightarrow$ W \subset \sim N_A −→< W||C >< W||C >−→< W|C >< W|C >

This is the formalism of DNA replication. This is the formalism of DIMI representer. This is the formalism of DNA replication.

with \mathcal{M} and an already-waiting strand \mathcal{M} . The pair with \mathcal{M} is pair with \mathcal{M} .

a member of X. Note the repetition of X. Note the repetition of X in the definition $\mathcal{X} = \mathcal{X} \cup \mathcal{X} = \mathcal{X} \cup \mathcal{X}$ Where is the repetition in the of R exactly when it is not the case that R is a member of R is the Russell R is the Russell R is the Russell DNA Self-Replication?

There is not space here to go into all these relationships, but the Russell

Where is the repetition in the DNA self-replication? The repetition and the replication are no longer separated. The repetition occurs not syntactically, but directly at the point of replication. Note the device of pairing or mirror imaging. A calls up the appearance of T and G calls up the appearance of $C \leq W$ calls up the appearance of $|C| >$ and $|C| >$ calls up the appearance of $\lt W$. Each object O calls up the appearance of its *dual or paired object* O^* . O calls up O^* and O^* calls up O. The object that replicates is implicitly a repetition in the form of a pairing of object and dual object.

OO[∗] −→ O O[∗] −→ OO[∗] OO[∗]

.

 \mathbf{r} canctition is inhorent in the replicand Γ checally the mind calls in the represent of Γ in the sense that the dual of a form \vert \bullet or \bullet and \bullet calls up O \bullet calls up O \bullet calls up O \bullet calls up O \bullet in plicates is implicitly up O \bullet is inherent in the form. The repetition is inherent in the replicand

OO[∗] replicates via

 $O \longrightarrow O O^*$

replication are no longer separated. The repetition occurs not syntactically,

but directly at the point of replication. Note the device of pairing or mirror

 $O^* \longrightarrow OO^*$

whence

 $OO^* \longrightarrow O O^* \longrightarrow OO^* O O^*.$

In this duality, O^* is the blueprint for OO* and O is also the blueprint for OO*.

 $\rm DNA = <>$ environment with its supply of available base pairs (here symbolized by the symbolized by the symbolized by th
Its supply of available by the symbolized by the symbolized by the symbolized by the symbolized by the symboli individual left and right brackets). Then when the DNA strategy \geq

 $\boxed{\text{DNA} = \texttt{<>>}\longrightarrow \texttt{<} E \texttt{>}\longrightarrow \texttt{<}> \texttt{=} \texttt{DNA} \text{ DNA}. }$ $DNA = \langle \rangle \longrightarrow \langle E \rangle \longrightarrow \langle \rangle \langle \rangle =$ DNA DNA.

encounter the matching bases from the environment and become two DNA's.

We regard the two brackets of the two brackets of the two brackets of the container as representatives for the
The two brackets of the two brackets of the two brackets for the two brackets for the two brackets for the two

matched DNA strands. We let the extrands. We let the extrands. We let the extrands. We let the cellular the ce
The cellular the ce

 $\mathsf E$ is the "environment". Life is the "environment".

 $\sum_{i=1}^n E_i$ is replaced by \ge . $\begin{array}{ccc} \text{F is replaced by } > < < \\ \end{array}$ $\mathcal{L} = \mathcal{L} \cdot \mathcal{L}$ is the automorphism in conjunction with the automorphism $\mathcal{L} = \mathcal{L} \cdot \mathcal{L}$ E is replaced by $><$.

 $\begin{array}{ccc} & \text{if} \leq > \text{ is a container,} \ & & \text{then } \geq \leq \text{ is an extainer.} \end{array}$ If <> is a container, then >< is an extainer.

 $\langle \rangle \langle \rangle = \langle \rangle \langle \rangle = \langle \rangle$ $\left\langle \right\rangle$ concepts of extra containers and containers and containers lead, in Section of extra containers lead, in Section of extra containers lead, in Section of extra containers and containers lead, in Section of the s $\langle \rangle \langle \rangle = \langle \rangle \langle \rangle = \langle \rangle$

6, to a new approach to the structure of and generalizations of the Temperley Lieb algebra. In this Section we discuss how projectors in the Temperley Lieb $>>$ $>>$ $=$ $>>$

 $\mathcal{L}=\mathcal{L}=\mathcal{L}+\mathcal{$

algebra can be regarded as topological/algebraic models of self-replication, algebraic models of self-replication, a
Algebraic models of self-replication, and the planet models of self-replication, and the planet models of

and we take this point of view take this point of view to characterize multiplicative elements \sim

Recursive Distinctioning

Recursive Distinctioning means just what it says. A pattern of distinctions is given in a space based on a graphical structure (such as a line of print or a planar lattice or given graph). Each node of the graph is occupied by a letter from some arbitrary alphabet. A specialized alphabet is given that can indicate distinctions about neighbors of a given node. The neighbors of a node are all nodes that are connected to the given node by edges in the graph. The letters in the specialized alphabet (call it SA) are used to describe the states of the letters in the given graph and at each stage in the recursion, letters in SA are written at all nodes in the graph, describing its previous state. The recursive structure that results from the iteration of descriptions is called Recursive Distinctioning.

Here is an example (Joel Isaacson). We use a line graph and represent it just as a finite row of letters. The Special Alphabet is $SA = \{ =, [,], O\}$ where "=" means that the letters to the left and to the right are equal to the letter in the middle. Thus if we had AAA in the line then the middle A would be replaced by $=$. The symbol "[" means that the letter to the LEFT is different. Thus in ABB the middle letter would be replaced by [. The symbol "]" means that

the letter to the right is different. And finally the symbol "O" means that the letters

both to the left and to the right are different. SA is a tiny language of elementary letter-distinctions.

Here is an example of this RD in operation where

we use the proverbial three dots to indicate a long string of letters in the same pattern. For example,

Note that the element [O] appears and it has replicated itself in a kind of mitosis.

RD = Recursive Distinguishing

RD Self-Replication is analogous to DNA Self-Replication.

(We explain below.)

Recursive Distinguishing

A letter will receive "[" if it is equal on the right and unequal on the left.

A letter will receive "]" if it is equal on the left and unequal on the right.

A letter will receive "O" if it is unequal on the left and unequal on the right.

...AAAAAAAABAAAAAAAA...

...= = = = = =] \bigcirc [= = = = = == = = = =] O O O [= = = == = = =] O [=] O [= = = ...

...AAAAAAAABAAAAAAAA... ...= = = = = =] O [= = = = = == = = = =] O O O [= = = = == = = =] O [=] O [= = = = ...

A single distinction (the letter B in the row of same A's) has been described and the description itself described two more times.

>]O[can be regarded in the pattern of DNA Replication.

In the context of recursive distinguishing, recursive redescription, a simple local distinction gives birth to an entity]O[that can reproduce itself!

Philosophically speaking, this is the whole talk. The RD process is fundamental and primordial.

On the Mathematical Side written in the form \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} and then then \mathcal{L} makes projection operators by recombining in the operators by \mathcal{A} { }

Each left or right bracket in itself makes a distinction. The two brackets are distinct from one another by mirror imaging, which we take to be a notational reflection of a fundamental process (of distinction) whereby two forms are identical (indistinguishable) except by comparison in the space of an observer. The observer is the distinction between the mirror images. Mirrored pairs of individual brackets interact to form either a container

 $C = \{\}$

or an extainer

 $E = \} \{$.

Two containers interact to form an extra containers interact to form an extra container brackets. Two containers σ

These new forms combine to make:

$$
CC = \{\}\} = \{E\}
$$

 $\overline{}$

and and

of this paper.

 $EE = \{\}\{\}=\}C\{$.

mind, it is natural to decide to make the container an analog of a scalar quan-

tity and let it commute with individual brackets. We then have the equation $\mathcal{M}(\mathcal{C})$

$$
EE = \{\{\}\{=\}C\} = C\} = CE.
$$

mind, it is natural to decide to make the container an analog of a scalar quan-

extainers interact to form a containers interact to form a container brackets. The pattern brackets. The pattern

of extainer interactions can be regarded as a formal generalization of the bra

and ket patterns of the Dirac notation that we have used in this paper both we have used in this paper both th
This paper both we have used in this paper both we have used in this paper both we have used in this paper bot

of extractions can be regarded as a formal generalizations can be regarded as a formal generalization of the br It is natural to make the container the analog of a scalar quantity. The analog of a scalar quantity and make it commute with $\overline{ }$ commuting scalar, while }{ corresponds to |A >< B |, a matrix that does not to individual brackets. $\frac{1}{\pi}$ axiom the containers computer $\frac{1}{\pi}$ $\begin{array}{ccc} \text{Circ array} & \text{Circ array} \\ \text{Circ array} & \text{Circ array} \end{array}$ and make it commute with $\begin{array}{c|c} \hline \end{array}$

point, we have described the basis for the basis for the formalism used in the formalism used in the earlier par

We can also regard $EE = \{\}E$ as symbolic of the emergence of DNA from the chemical substrate. Just as the formalism for reproduction ignores the topology, this formalism for emergence ignores the formation of the DNA backbone along which are strung the complementary base pairs. In the biological domain we are aware of levels of ignored structure.

Arising From a Substrate of Rules and Interactions Self-Reference and Cell Self-Assembly **Entity** Linkage **Catalyst** Autopoesis.

In the course of time the catalysts (basically separate from one another due to lack of bonding) become surrounded by circular forms of bonded or partially bonded substrate. A distinction (in the eyes of the observer) between inside (near the catalyst) and outside (far from a given catalyst) has spontaneously arisen through the "chemical rules". Each catalyst has become surrounded by a proto-cell. No higher organism has formed here, but there is a hint of the possibility of higher levels of organization arising from a simple set of rules of interaction. The system is not programmed to make the proto-cells. They arise spontaneously in the evolution of the structure over time.

Fig. 1. The first seven instants (9-6) of one computer run, showing the spontaneous generation of an autopoletic unity. Interactions between substrate O and catalyst + produce chains of bonded links \boxtimes , which eventually enclose the catalyst, thus closing a network of interactions which constitutes an autopoietic unity within this universe.

In this 1,2,3 system of description, where is the selfreference?

 \bullet \bullet

22 describes itself.

 $• • •$

 $• • •$

We have looked at self-replication from logical, descriptive, recursive and biological points of view.

Key aspects of coding and reference occur with different emphasis, and we see that in each domain the intertwining of syntax and semantics takes a different form in the relation of the way that a universe divides into observer and observed.

Thank you for your attention.

\mathbf{r} are complex numbers are to a common multiple. States are to a common multiple. States are \mathbf{r} APPENDIX - Quantum Formalism $\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$. Thus $\overline{}$ $\sum_{i=1}^n$ in the subject $\sum_{i=1}^n$ is identified with the vector B $\sum_{i=1}^n$ $\begin{array}{ccc} \hbox{for all } & \mathbf{0} \end{array}$ dual element to A correspond to A correspond to A consider the contract to the contract transpose A contract t

 $\mathcal{L} = \mathcal{L} \times \mathcal{L}$

Dirac [5] introduced the "bra -(c)-ket" notation $\langle A|B \rangle = A^*B$ for the
inner product of complex vectors $\langle A, B \rangle \subset H$, He also separated the parts of inner product of complex vectors $A, B \in H$. He also separated the parts of the bracket into the kets- $\sum_{n=1}^{\infty} A_n A_n B_n$. Thus the bracket into the $bra < A$ and the ket $|B>$. Thus the bracket into the *bra* $\langle A \rangle$ and the *ket* $|B \rangle$. Thus the bracket into the $bra < A$ and the ket $|B>$. Thus Dirac [5] introduced the "bra -(c)-ket" notation $\langle A | B \rangle = A * B$ for the

 $\label{eq:3} = \right.$

Dirac can write the "ket bra" $|A| < E | = A B^*$ Dirac can write the "ket-bra" $|A\rangle \langle B| = AB^*$. conventional notation, the ket-bra is a matrix, not a scalar, and we have the $\mathcal{P}_2 = \mathcal{P}_2 = \mathcal{P}_1$

 $\mathcal{G} = \{ \mathcal{G} \mid \mathcal{G} \in \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \mathcal{G} \text{ is a finite number of } \mathcal{G} \text{ and } \math$

inner product of complex vectors A, B ∈ H. He also separated the parts of complex vectors A, B ∈ He also separa

the bracket into the bracket into the bracket into the bracket into the ket \mathcal{A}

product A∗B (which is a scalar since B is a scalar since B is a scalar since B is a column vector). Having sepa

conventional notation, the ket-bra is a matrix, not a scalar, and we have the

The standard example is a ket-bra \mathcal{A} , \mathcal{A} and \mathcal{A} are \mathcal{A} is a \mathcal{A} -axis that \mathcal{A}

single state in the state in the
International contract in the state in the st

Written entirely in Dirac notation we have not

 $P = |A > < B|$ $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ since $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ and $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ and $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ and $\mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ $\mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L}$

$$
P^2 = |A\rangle \langle B||A\rangle \langle B| = |A\rangle \langle B|A\rangle \langle B|
$$

 $=*B* |*A* > |*A* > *B*| = *B* |*A* > *P*.$ $=$ $\langle D \, | A \rangle$ $|A \rangle$ $\langle D \, | A \rangle$ $=< B | A > | A > < B | = < B | A > P.$

 $\mathcal{L} = \mathcal{L} \times \mathcal{L}$

\mathbb{P} = P. Then \mathbb{P} is a projection matrix, projection matrix, projection \mathbb{P} Sum over Paths (Possibilities)

If $\{|C_1>, |C_2>, \cdots |C_n>\}\$ is an orthonormal basis for H, and $P_i = |C_i><\}$ C_i , then for any vector $|A| >$ we have

$$
|A> = |C_1> + \cdots + |C_n>.
$$

Hence

$$
\langle B | A \rangle = \langle C_1 | A \rangle \langle B | C_1 \rangle + \dots + \langle C_n | A \rangle \langle B | C_n \rangle
$$

=
$$
\langle B | C_1 \rangle \langle C_1 | A \rangle + \dots + \langle B | C_n \rangle \langle C_n | A \rangle
$$

=
$$
\langle B | [C_1 \rangle \langle C_1 | + \dots + | C_n \rangle \langle C_n |] | A \rangle
$$

=
$$
\langle B | 1 | A \rangle.
$$

 $\mathcal{L}=\mathcal{L}(\mathcal{L}^{(k)})$

 $\frac{1}{2}$

$\sum_{k=1}^{n} P_k = \sum_{k=1}^{n} |C_k\rangle \langle C_k| = 1$

We have written this sequence of equalities from \mathcal{A} to \mathcal{A} and \mathcal{A} to \mathcal{A} to \mathcal{A}

emphasize the role of the identity of the identity

this event is equal to \mathbb{R}^n , the event is equal to \mathbb{R}^n , the event is equal to \mathbb{R}^n

so that one can write the
See the can write the can from C_i to B and that the intermediate states $|C_i\rangle$ are a complete set of orthonormal alternatives then we can assume that $\langle C_i | C_i \rangle = 1$ for each i is the sum of the probabilities of an arbitrary state being projected into one of these intermediate states. In the quantum context one may wish to consider the probability of starting in state $|A\rangle$ and ending in state $|B\rangle$. The square of the probability for this event is equal to $| \langle B | A \rangle|^2$. This can be refined if we have more knowledge. If it is known that one can go from A to C_i $(i = 1, \dots, n)$ and orthonormal alternatives then we can assume that $\langle C_i | C_i \rangle = 1$ for each i and that $\Sigma_i|C_i\rangle\langle C_i|=1$. This identity now corresponds to the fact that 1 of these intermediate states.

in state $|\mathcal{A}|$ and end in state $|\mathcal{A}|$. The state $|\mathcal{A}|$. The square of the probability for t

knowledge. If it is known that one can go from A to Cⁱ (i = 1, · · · , n) and

from Ci to B and that the intermediate states $\mathcal{L}_{\mathcal{A}}$ are a complete states $\mathcal{L}_{\mathcal{A}}$

². This can be refined if we have more

We compare

$$
E = |C>
$$

and

$$
1 = \sum_k |C_k\rangle \langle C_k|.
$$

That the unit 1 can be written as a sum over the intermediate states is an expression of how the environment (in the sense of the space of possibilities) impinges on the quantum amplitude, just as the expression of the environment as a soup of bases ready to be paired (a classical space of possibilities) serves as a description of the biological environment. The symbol $E = |C| \lt U |$ indicated the availability of the bases from the environment to form the complementary pairs. The projection operators $|C_i\rangle \langle C_i|$ are the possibilities for interlock of initial and final state through an intermediate possibility. In the quantum mechanics the special pairing is not of bases but of a state and a possible intermediate from a basis of states. It is through this common theme of pairing that the conceptual notation of the bras and kets lets us see a correspondence between such separate domains.

 $\mathcal{F}_{\mathcal{F}}$ in all $\mathcal{F}_{\mathcal{F}}$ is not possible to contract it is not possible to copy a quantum mechanical $\mathcal{F}_{\mathcal{F}}$

Proof of the No Cloning Theorem. In order to have a quantum process make a copy of a quantum state we need a unitary mapping $U : H \otimes H \longrightarrow$ $H \otimes H$ where H is a complex vector space such that there is a fixed state $|X\rangle \in H$ with the property that

$$
U(|X > |A >) = |A > |A >
$$

for any state $|A\rangle \in H$. $(|A\rangle |B\rangle)$ denotes the tensor product $|A\rangle \otimes |B\rangle$. Let

 $T(|A>) = U(|X > |A >) = |A > |A >$.

Note that T is a linear function of $|A>$. Thus we have

 $T|0 \rangle = |0 \rangle |0 \rangle = |00 \rangle,$

 $T|1>=|1>|1>=|11>,$

$$
T(\alpha|0>+\beta|1>) = (\alpha|0>+\beta|1>)(\alpha|0>+\beta|1>).
$$

But

 $T(\alpha|0\rangle + \beta|1\rangle) = \alpha|00\rangle + \beta|11\rangle.$

Hence

$$
\alpha|00 \rangle + \beta|11 \rangle = (\alpha|0 \rangle + \beta|1 \rangle)(\alpha|0 \rangle + \beta|1 \rangle)
$$

$$
= \alpha^2 |00 \rangle + \beta^2 |11 \rangle + \alpha \beta |01 \rangle + \beta \alpha |10 \rangle
$$

From this it follows that $\alpha\beta = 0$. Since α and β are arbitrary complex numbers, this is a contradiction.

The proof of the no-cloning theorem depends constant theorem depends constant $\mathcal{L}_\mathbf{z}$

The proof of the no-cloning theorem depends crucially on the linear superposition of quantum states and the linearity of quantum process. By the time we reach the molecular level and attain the possibility of copying DNA molecules we are copying in a quite different sense than the ideal quantum copy that does not exist. The DNA and its copy are each quantum states, but they are different quantum states! That we see the two DNA molecules as identical is a function of how we filter our observations of complex and entangled quantum states. Nevertheless, the identity of two DNA copies is certainly at a deeper level than the identity of the two letters "i" in the word identity. The latter is conventional and symbolic. The former is a matter of physics and biochemistry.