BIOLOGIC-
Replication and Self-Reference
in
Formal Systems and Biology

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What is the relationship between logic,language,computation and biology?

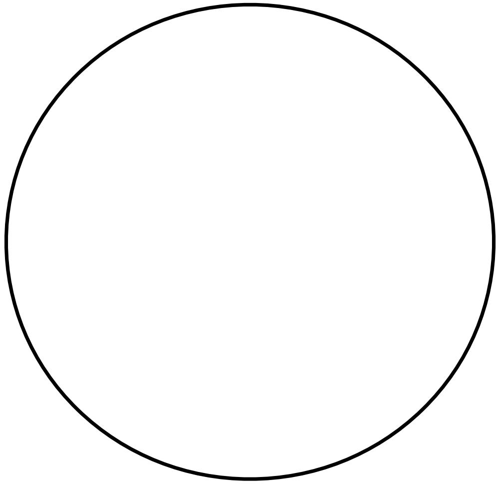
Classical Aspects:

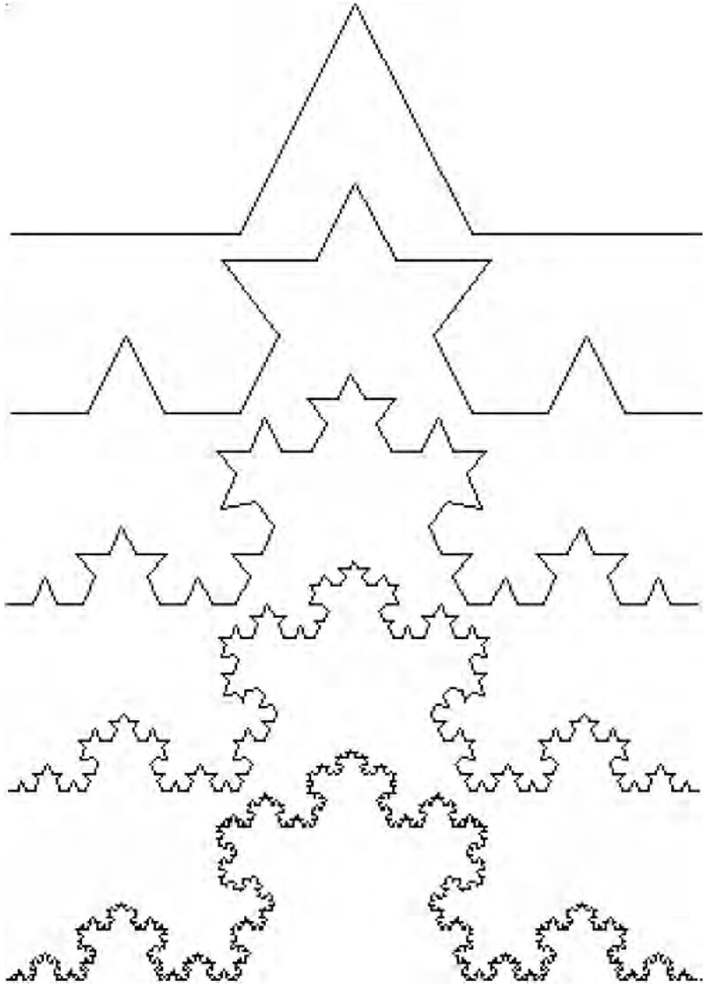
Self-Reference,
Recursion,
Imaginary Values.

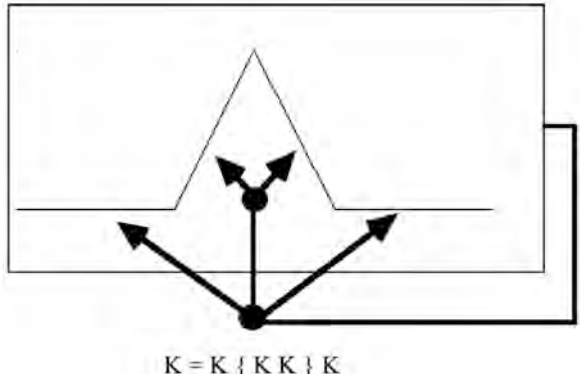
Symbols and
reproducibility of
symbols.

Separation of
object and reference.

A circle (intended to refer to distinction)
can be regarded as
referring to itself
as a
distinction.



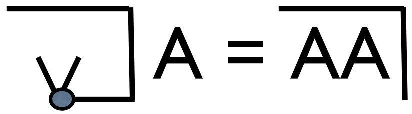


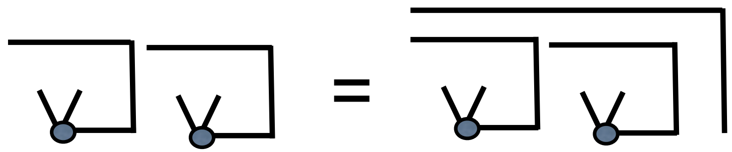


K = K{K K}K

The Framing of
Imaginary Space.

Fixed Point and Self-Replication





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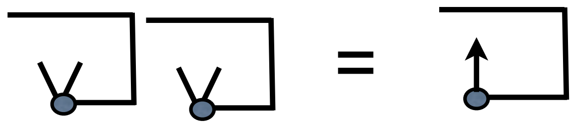
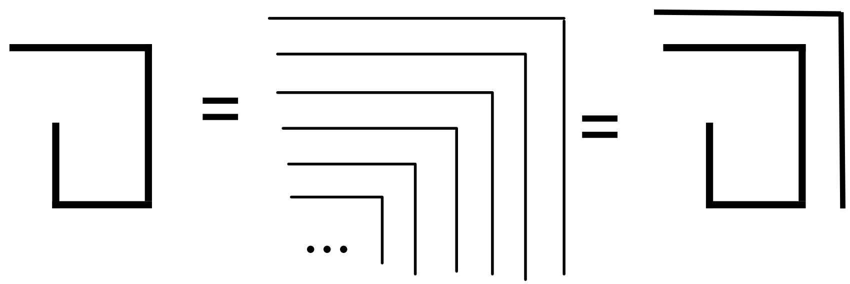
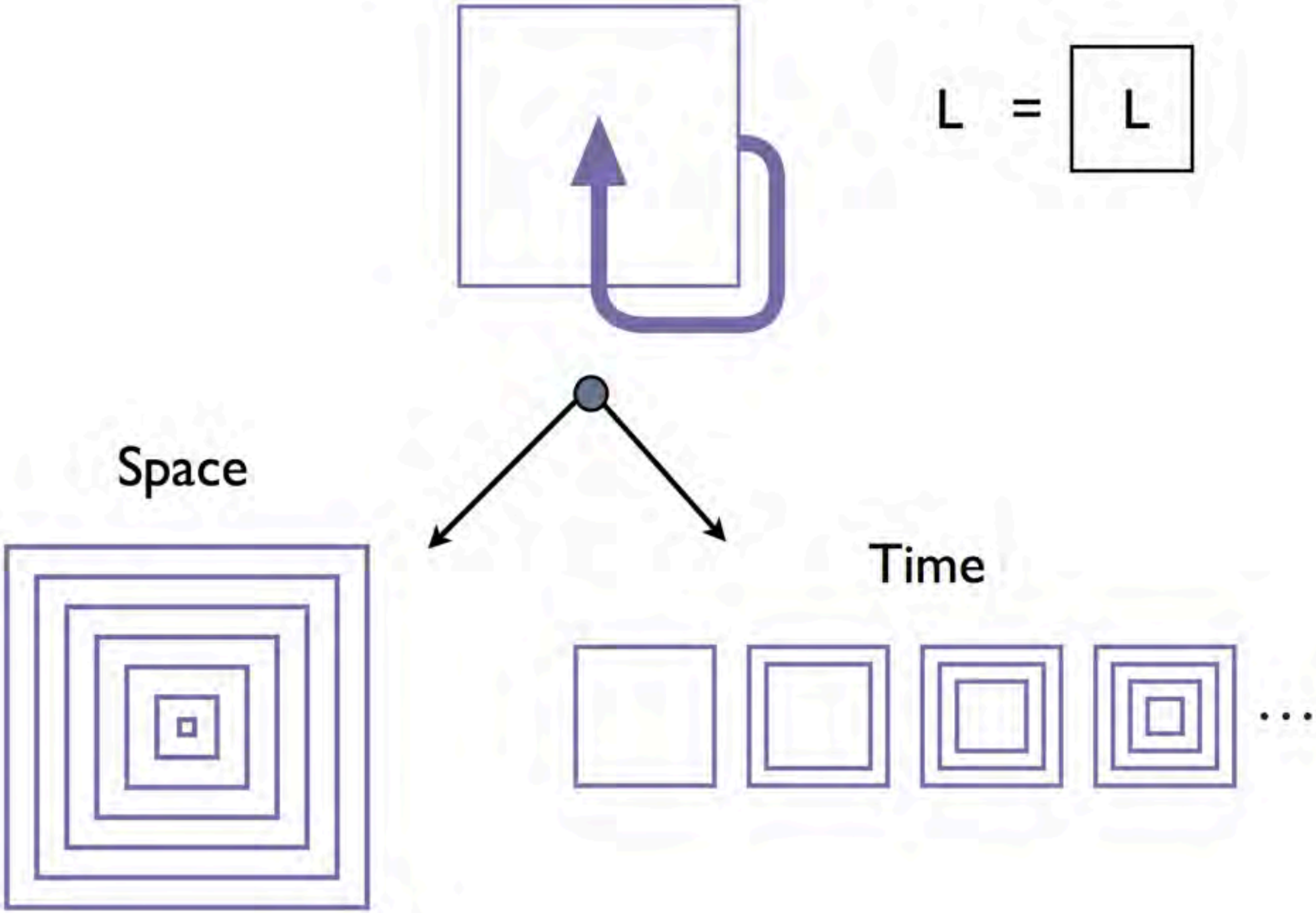




Recursion and Re-entry



Church-Curry Fixed Point Theorem.

In a reflexive domain any element
has a fixed point.

$gx = F(xx)$

$\downarrow$

$gg = F(gg)$

In a reflexive domain D
every element is an operator
on the domain D.

$D \longleftrightarrow [D,D]$

The von-Neumann Building Machine
can build itself.

B,x ⟶ B,x X,x

(x is the blueprint for X)

Let b be the blueprint for B.

Then B,b builds itself.

B,b ⟶ B,b B,b

Indicative Shift

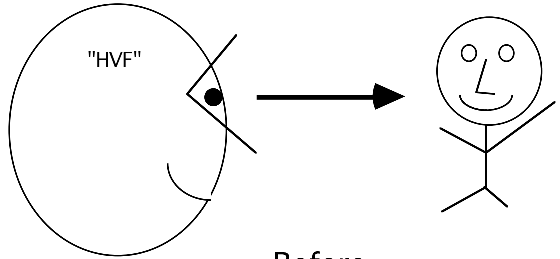
(name) (named)

A ⟶ B

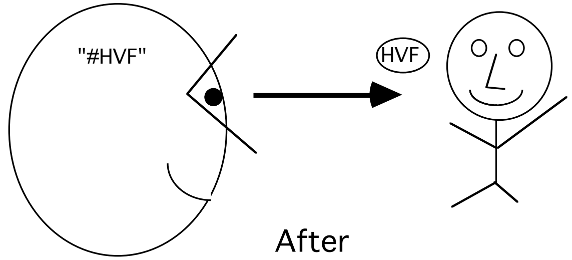
"A refers to B."

Then

#A ⟶ BA



Before



After

Suppose that M ⟶ #.

Then #M ⟶ #M. self-reference

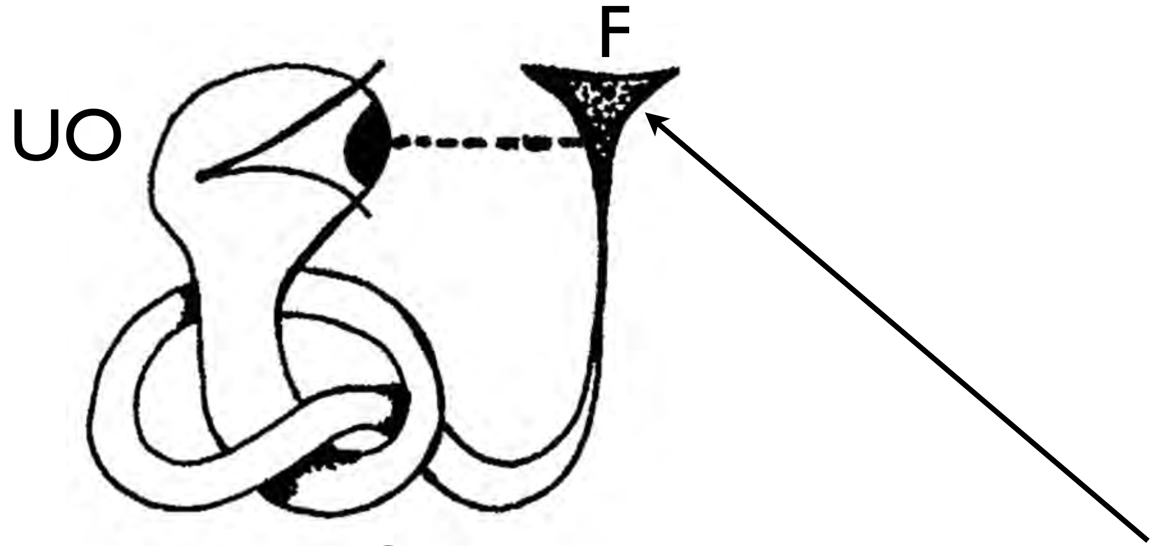
And if g ⟶ F#,
then #g ⟶ F#g. Godelian self-reference

Goedelian Reference

(code number)		(formula)
g	→	F(u)
#g	→	F(g)
g	→	~B(#u)
#g	→	~B(#g)

~B(#g) asserts its own unprovability.

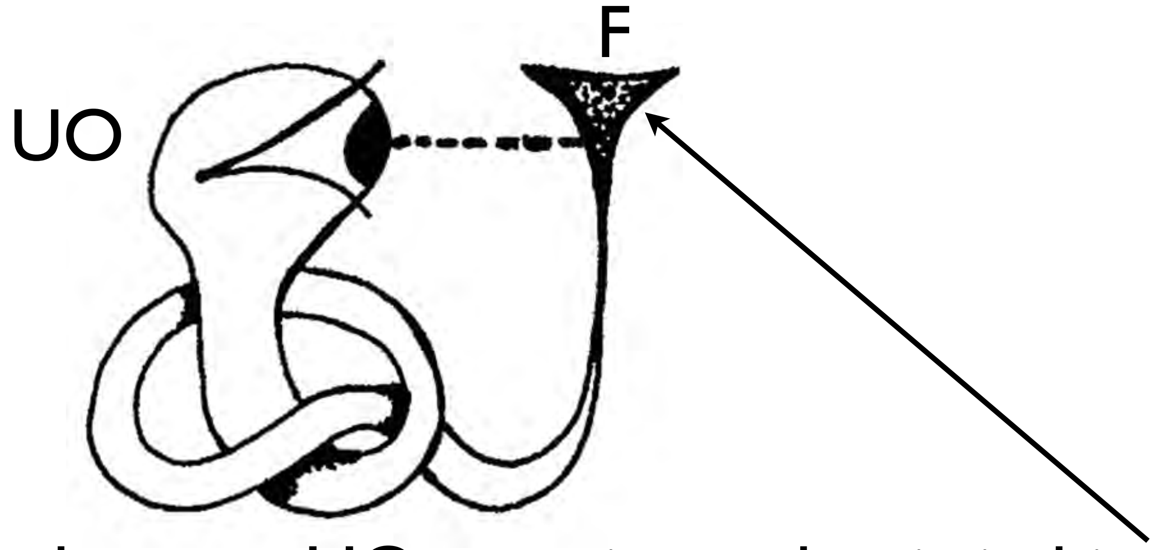
Note that the incompleteness phenomenon does not occur at level of indicative shift. It occurs at the cut between formal system and observer of formal system.



A universal observer UO examines what is in his hand.

He sees a formal system F and wonders if perhaps F is the complete model of his ability to reason.

But no, it cannot be! For UO can prove the incompleteness of F and so UO knows that he is not identical with F, just so long as UO and F are both consistent in their reason.



A universal observer UO examines what is in his hand.

The UOF is not a UFO, but she is beyond Turing. Is the necessary cognition for this ability related to biology and/or (Penrose) new physics?

$$M \longrightarrow \#$$

$$\# M \longrightarrow \# M$$

Self Reference occurs at the Shift
of the Name M of the
Meta-Naming Operator #.

" I am the
Observed relation
Between myself
And
Observing myself."

(Heinz von Foerster)

In a Nutshell:

Rx ---------> xx

then

RR---------->RR

and

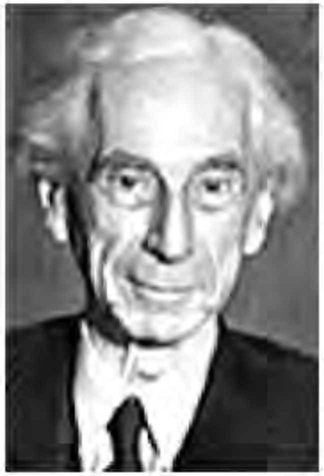
Beware the Jabberwock!

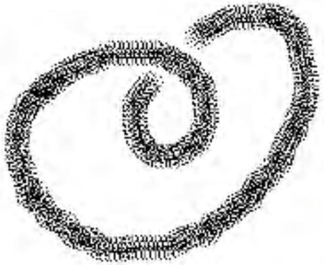
Rx = ~xx

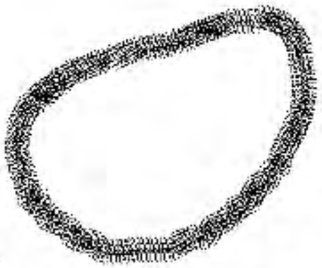
RR = ~RR

$$Rx = \sim xx$$
$$RR = \sim RR$$

Russell Paradox (K)not.





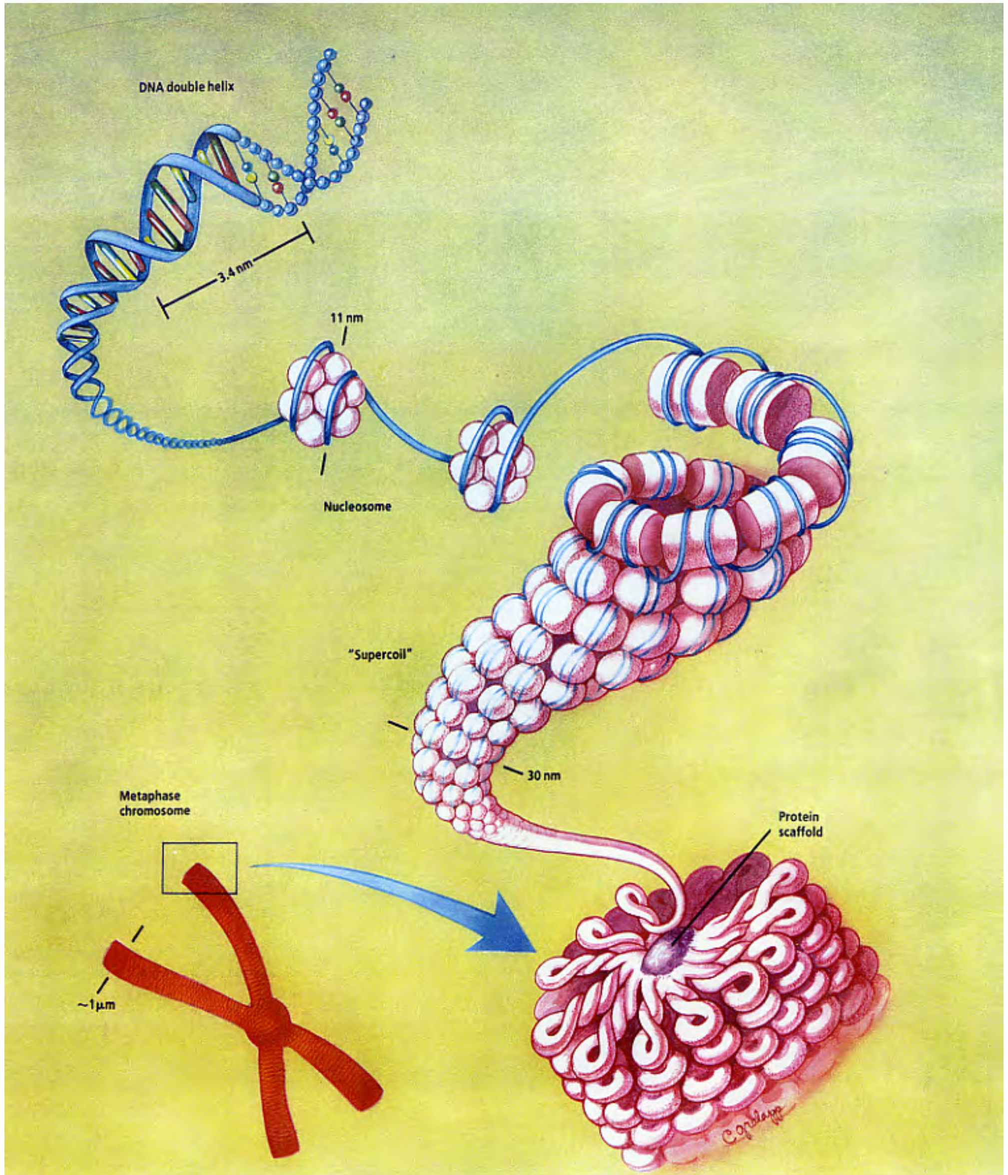


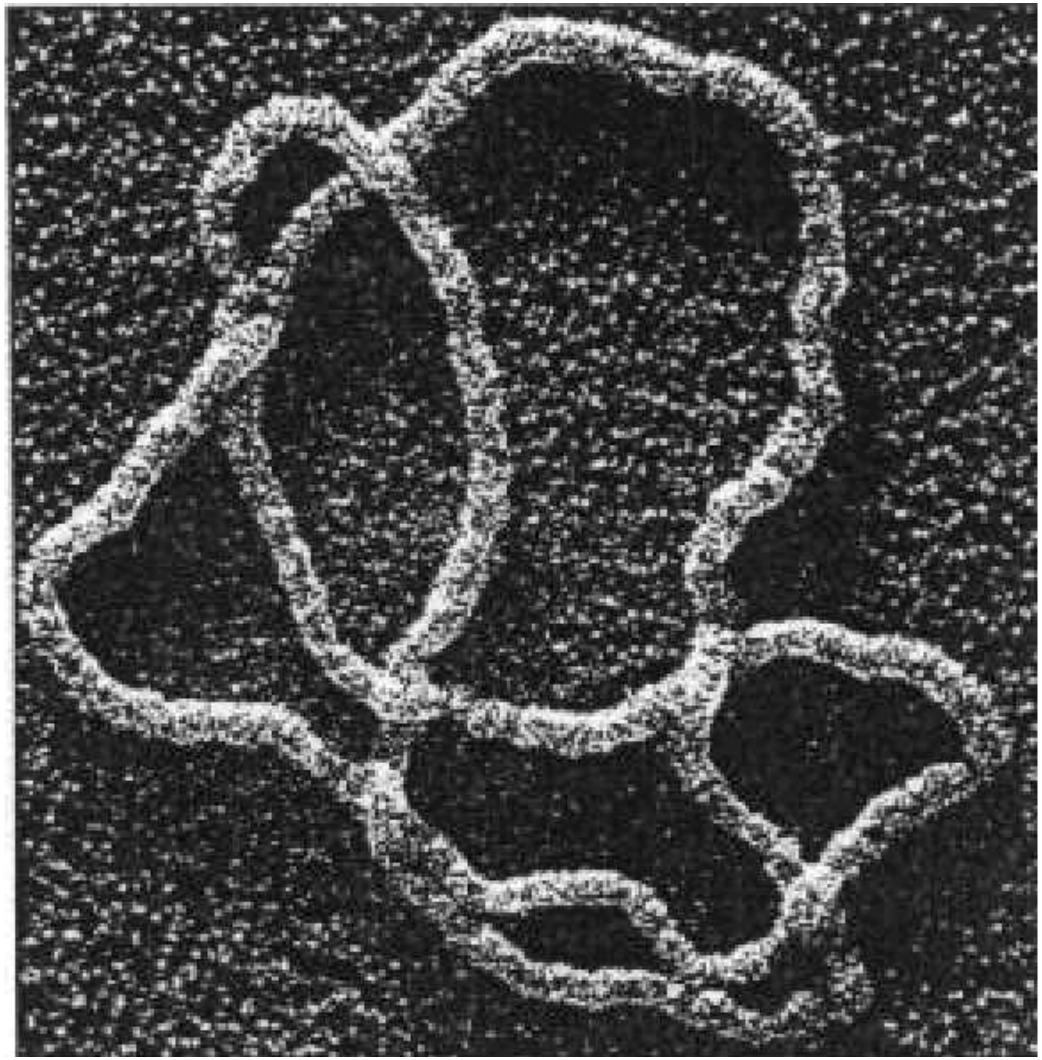
A belongs to A. ⟺ A does not belong to A.

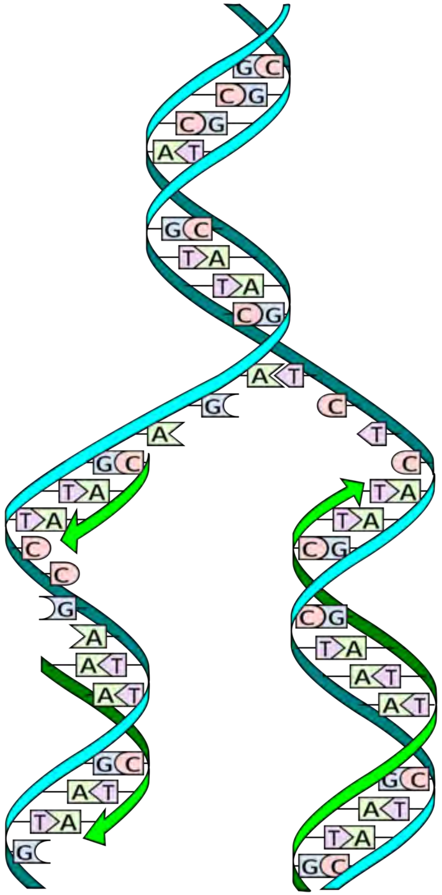
So far, this is the story of the
classical logic of self-replication
and self-reference.

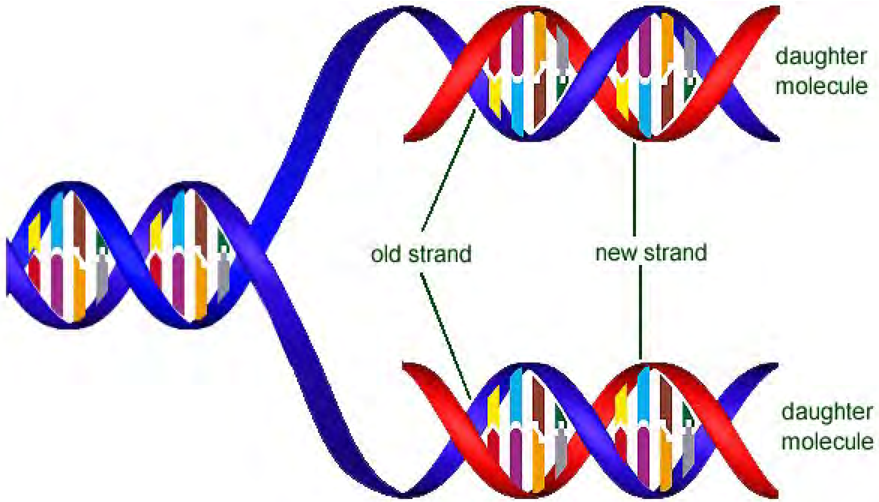
We know that DNA engages
in self-replication.

How does the DNA self-rep
compare with our
familiar self-replication
at the level of logic and
recursion?

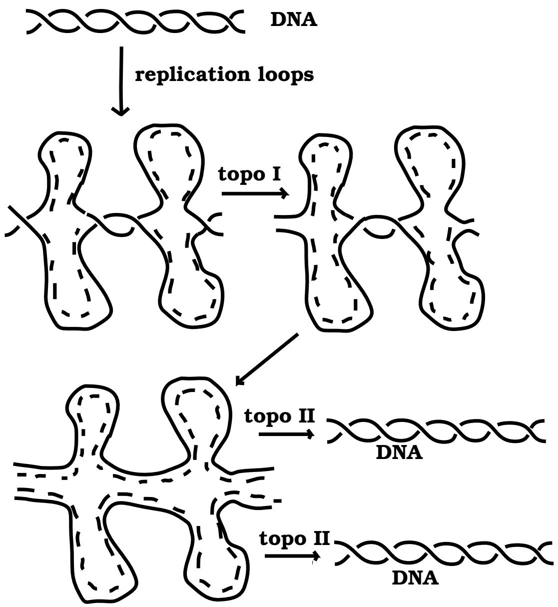

DNA double helix
3.4 nm
11 nm
Nucleosome
"Supercoil"
30 nm
Metaphase chromosome
~1µm
Protein scaffold






daughter molecule
old strand
new strand
daughter molecule

DNA is a Self-Replicating Form



$$DNA = < W|C >$$

$$< W| = < ...TTAGAATAGGTACGCG...|$$

$$|C >= |...AATCTTATCCATGCGC... > .$$

$$< W| + E \longrightarrow < W|C >= DNA$$

$$E + |C > \longrightarrow < W|C >= DNA$$

$$< W|C > \longrightarrow < W| + E + |C >=< W|C >< W|C >$$

Self Replication Schematic

DNA = <Watson|Crick>

E = Environment

The base pairs are AT (Adenine and Thymine) and GC (Guanine and Cytosine). Thus if

$$< W| =< ...TTAGAATAGGTACGCG...|$$

Then

$$|C >= |...AATCTTATCCATGCGC... > .$$

Symbolically we can oversimplify the whole process as

$$< W| + E \longrightarrow < W|C >= DNA$$

$$E + |C > \longrightarrow < W|C >= DNA$$

$$< W|C > \longrightarrow < W| + E + |C >=< W|C >< W|C >$$

This is the formalism of DNA replication.

Where is the repetition in the DNA Self-Replication?

Where is the repetition in the DNA self-replication? The repetition and the replication are no longer separated. The repetition occurs not syntactically, but directly at the point of replication. Note the device of pairing or mirror imaging. A calls up the appearance of T and G calls up the appearance of C. $< W|$ calls up the appearance of $|C >$ and $|C >$ calls up the appearance of $< W|$. Each object O calls up the appearance of its *dual or paired object* O^*. O calls up O^* and O^* calls up O. The object that replicates is implicitly a repetition in the form of a pairing of object and dual object.

The repetition is inherent in the replicand in the sense that the dual of a form is inherent in the form.

OO^* replicates via

$$O \longrightarrow OO^*$$

$$O^* \longrightarrow OO^*$$

whence

$$OO^* \longrightarrow O \;\; O^* \longrightarrow OO^* \;\; OO^*.$$

In this duality, O* is the blueprint for OO* and O is also the blueprint for OO*.

$\mathrm{DNA} = <>$

$$\mathrm{DNA} = <> \longrightarrow < E > \longrightarrow <><> = \ \mathrm{DNA\ DNA}.$$

E is the "environment".

E is replaced by ><.

If <> is a container,
then >< is an extainer.

<><> = < >< >

>< >< = ><><

DNA Self-Replication Schema

DNA = <Watson|Crick>

$\longrightarrow$ <Watson| Environment | Crick>

$\longrightarrow$ <Watson| |Crick><Watson| | Crick>

$\longrightarrow$ <Watson|Crick><Watson|Crick>

$\longrightarrow$ DNA DNA

The DNA divides into its own blueprints for replication.

Recursive Distinctioning

Recursive Distinctioning means just what it says. A pattern of distinctions is given in a space based on a graphical structure (such as a line of print or a planar lattice or given graph). Each node of the graph is occupied by a letter from some arbitrary alphabet. A specialized alphabet is given that can indicate distinctions about neighbors of a given node. The neighbors of a node are all nodes that are connected to the given node by edges in the graph. The letters in the specialized alphabet (call it SA) are used to describe the states of the letters in the given graph and at each stage in the recursion, letters in SA are written at all nodes in the graph, describing its previous state. The recursive structure that results from the iteration of descriptions is called Recursive Distinctioning.

Here is an example (Joel Isaacson) . We use a line graph and represent it just as a finite row of letters. The Special Alphabet is SA = { =, [,], O} where "=" means that the letters to the left and to the right are equal to the letter in the middle. Thus if we had AAA in the line then the middle A would be replaced by =. The symbol "[" means that the letter to the LEFT is different. Thus in ABB the middle letter would be replaced by [. The symbol "]" means that the letter to the right is different. And finally the symbol "O" means that the letters both to the left and to the right are different. SA is a tiny language of elementary letter-distinctions. Here is an example of this RD in operation where we use the proverbial three dots to indicate a long string of letters in the same pattern. For example,

```
... AAAAAAAAAABAAAAAAAAAA ... is replaced by
... =========]O[========= ... is replaced by
... ========]OOO[======== ... is replaced by
... =======]O[=]O[======= ... .
```

Note that the element]O[appears and it has replicated itself in a kind of mitosis.

RD = Recursive Distinguishing

RD Self-Replication
is analogous to
DNA Self-Replication.

1. $=======]O[=======\ .$

2. $======]OOO[=======$

3. $=====]O[=]O[======$

(We explain below.)

Recursive Distinguishing

A letter will receive “[“ if it is equal on the right and unequal on the left.

A letter will receive “]“ if it is equal on the left and unequal on the right.

A letter will receive “O“ if it is unequal on the left and unequal on the right.

...AAAAAAAABAAAAAAAA...

...= = = = = =] O [= = = = = = ...

...= = = = =] O O O [= = = = = ...

...= = = =] O [=] O [= = = = ...

```
*AAAAAAAAAAAAAAAAABAAAAAAAAAAAAAAAAA*
*                 ]O[                 *
*                ]OOO[                *
*               ]O[ ]O[               *
*              ]OOOOOOO[              *
*             ]O[     ]O[             *
*            ]OOO[   ]OOO[            *
*           ]O[ ]O[ ]O[ ]O[           *
*          ]OOOOOOOOOOOOOOO[          *
*         ]O[             ]O[         *
*        ]OOO[           ]OOO[        *
*       ]O[ ]O[         ]O[ ]O[       *
*      ]OOOOOOO[       ]OOOOOOO[      *
*     ]O[     ]O[     ]O[     ]O[     *
*    ]OOO[   ]OOO[   ]OOO[   ]OOO[    *
*   ]O[ ]O[ ]O[ ]O[ ]O[ ]O[ ]O[ ]O[   *
*  ]OOOOOOOOOOOOOOOOOOOOOOOOOOOOOOO[  *
* ]O[                             ]O[ *
```

...AAAAAAAABAAAAAAAA...

...= = = = = =] O [= = = = = = ...

...= = = = =] O O O [= = = = = ...

...= = = =] O [=] O [= = = = ...

A single distinction (the letter B in the row of same A's) has been described and the description itself described two more times.

]O[can be regarded in the pattern of DNA Replication.

In the context of recursive distinguishing, recursive re-description, a simple local distinction gives birth to an entity]O[that can reproduce itself!

...= = = = = =] O [= = = = = = ...

...= = = = =] O O O [= = = = = ...

...= = = =] O [=] O [= = = = ...

Philosophically speaking, this is the whole talk.
The RD process is fundamental and primordial.

On the Mathematical Side

$\{ \quad \}$

Each left or right bracket in itself makes a distinction. The two brackets are distinct from one another by mirror imaging, which we take to be a notational reflection of a fundamental process (of distinction) whereby two forms are identical (indistinguishable) except by comparison in the space of an observer. The observer *is* the distinction between the mirror images. Mirrored pairs of individual brackets interact to form either a *container*

$$C = \{\}$$

or an *extainer*

$$E = \}\{.$$

These new forms combine to make:

$$CC = \{\}\{\} = \{E\}$$

and

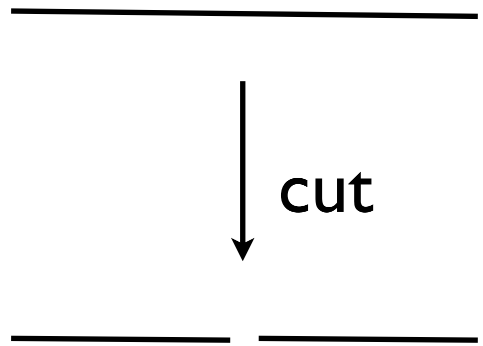
$$EE = \}\{\}\{ = \}C\{.$$

$$EE = \}\{\}\{ = \}C\{ = C\}\{ = CE.$$

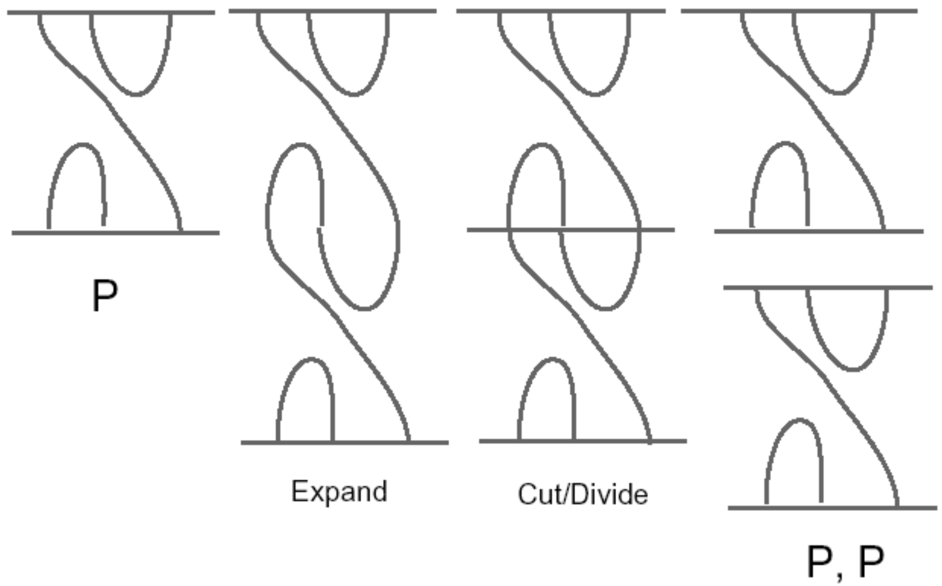
It is natural to make the container
the analog of a scalar quantity
and make it commute with
individual brackets.

We can also regard $EE = \{\}E$ as symbolic of the emergence of DNA from the chemical substrate. Just as the formalism for reproduction ignores the topology, this formalism for emergence ignores the formation of the DNA backbone along which are strung the complementary base pairs. In the biological domain we are aware of levels of ignored structure.

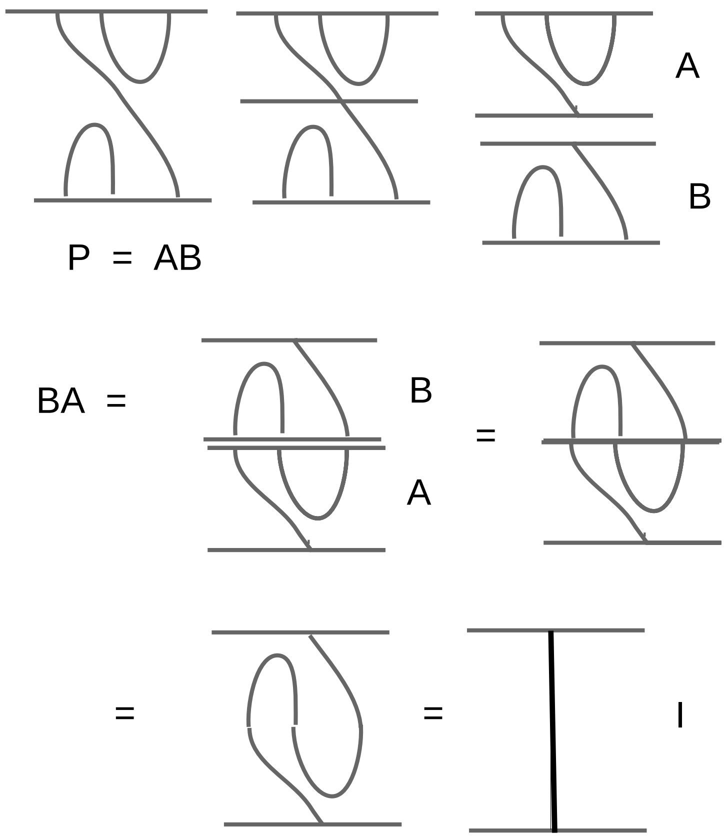
Simplest Replication



Topological Replication



Why the topological self-rep worked.



BA = I

So P = AB = AIB = ABAB = PP.

Autopoesis.
Self-Reference and Cell Self-Assembly
Arising From a Substrate of
Rules and Interactions
Entity
Linkage
Catalyst

In the course of time the catalysts (basically separate from one another due to lack of bonding) become surrounded by circular forms of bonded or partially bonded substrate. A distinction (in the eyes of the observer) between inside (near the catalyst) and outside (far from a given catalyst) has spontaneously arisen through the "chemical rules". Each catalyst has become surrounded by a proto-cell. No higher organism has formed here, but there is a hint of the possibility of higher levels of organization arising from a simple set of rules of interaction. *The system is not programmed to make the proto-cells.* They arise spontaneously in the evolution of the structure over time.

BioSystems 5 (1974) 187–196. NORTH-HOLLAND PUBLISHING COMPANY, AMSTERDAM

AUTOPOIESIS: THE ORGANIZATION OF LIVING SYSTEMS, ITS CHARACTERIZATION AND A MODEL

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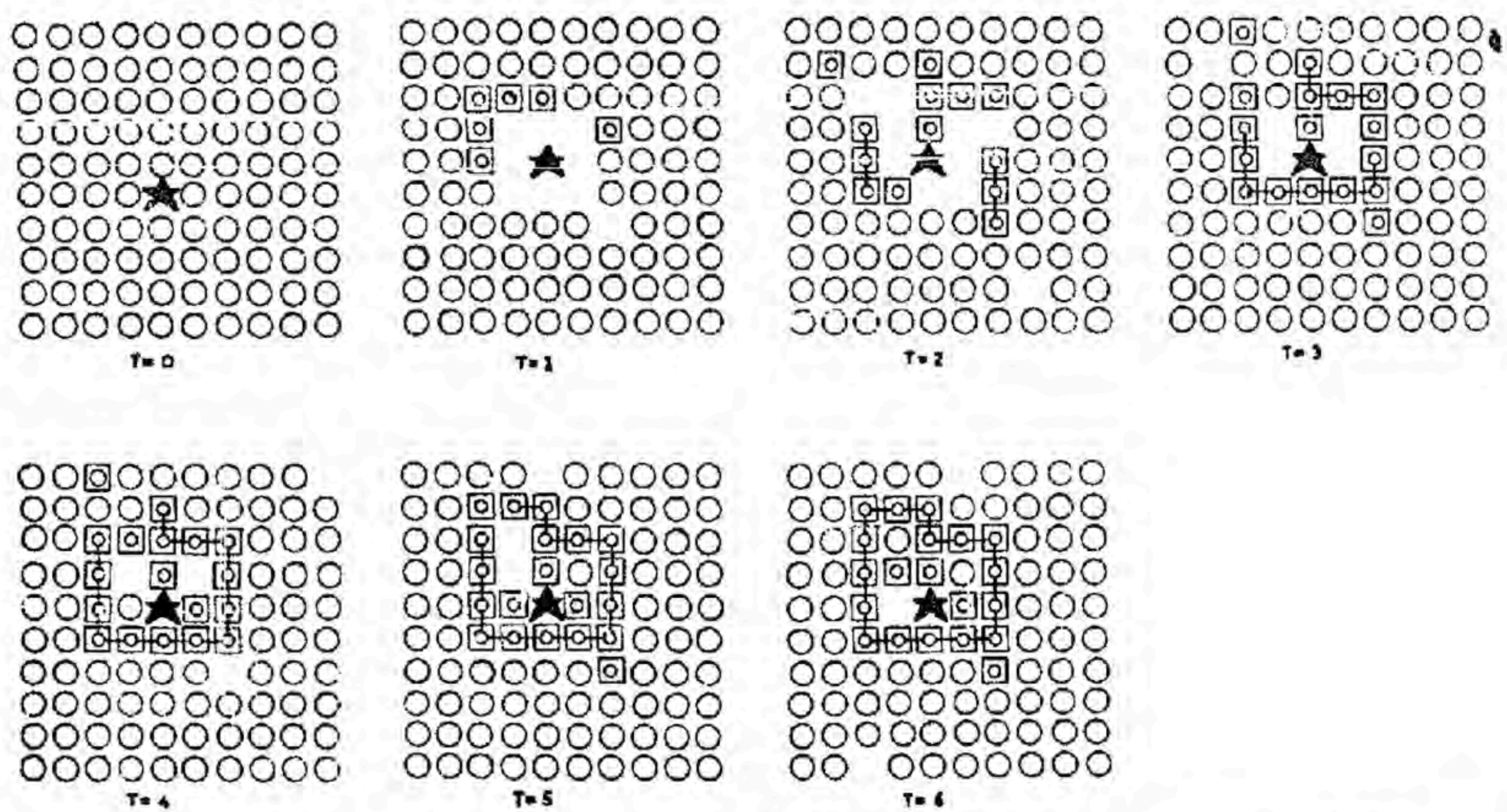


Fig. 1. The first seven instants (0→6) of one computer run, showing the spontaneous generation of an autopoietic unity. Interactions between substrate ○ and catalyst ★ produce chains of bonded links ⊡, which eventually enclose the catalyst, thus closing a network of interactions which constitutes an autopoietic unity within this universe.

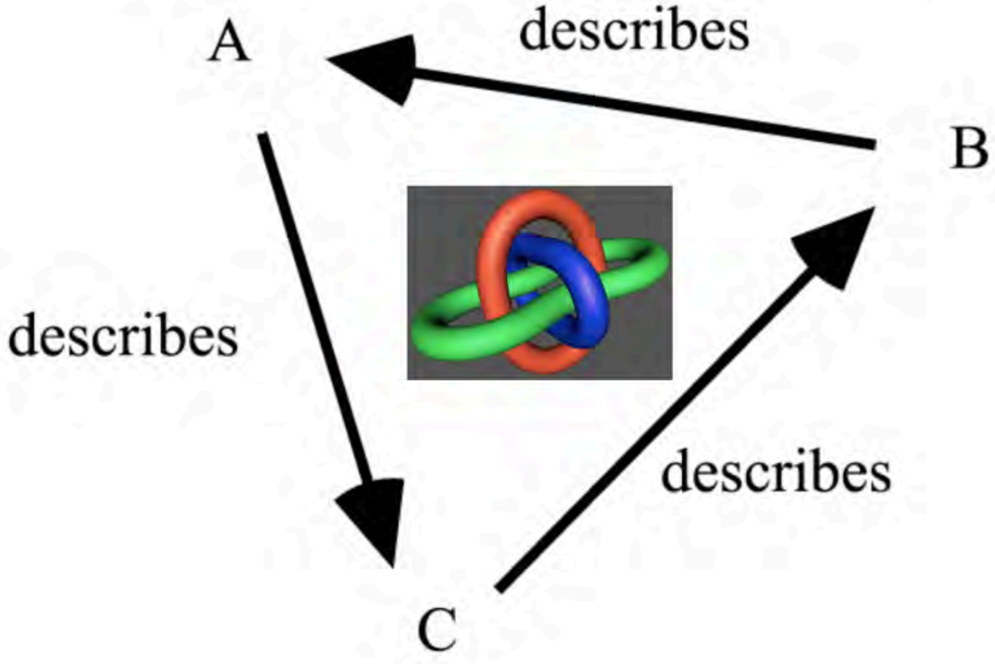
Describing Describing

3
13
1113
3113
132113
1113122113
...

$$\mathbf{A} = \mathbf{11131221131211132221}\ldots$$

$$\mathbf{B} = \mathbf{3113112221131112311332}\ldots$$

$$\mathbf{C} = \mathbf{132113213221133112132123}\ldots$$



In this 1,2,3 system of description, where is the self-reference?

33	xx	22
23	2x	22
1213	121x	22
...	...	...

22 describes itself.

Topological Processes

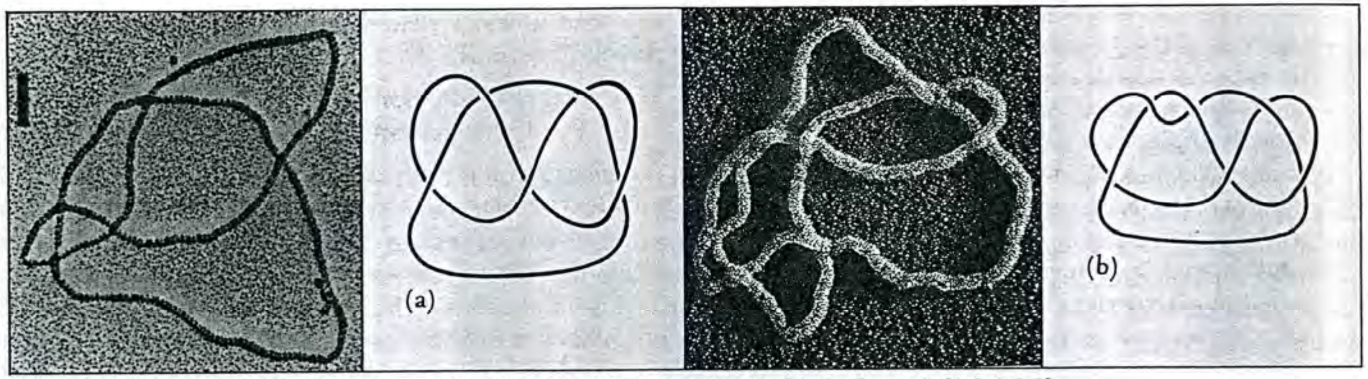
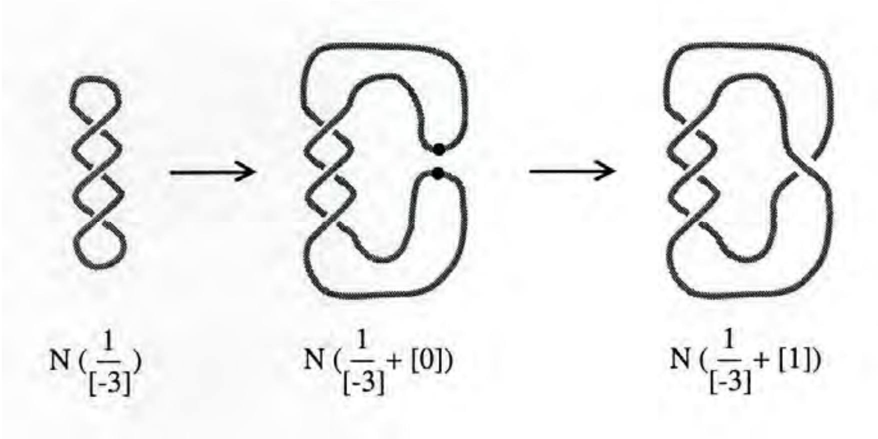


Figure 8. DNA 4-plats (Tn3) (a) shows the Whitehead link [1,1,1,1,1]; (b) the knot 6_2 [1,2,1,1,1].

DNA Recombination



Tangle Model: Ernst & Sumners, 1989

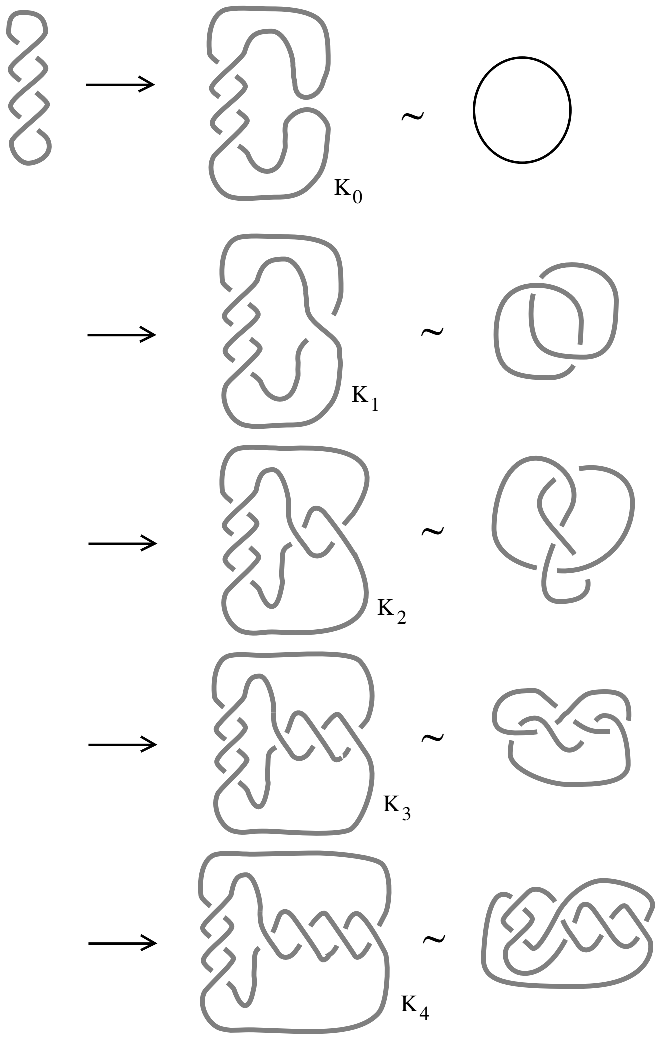


Figure 28 - Processive Recombination with $S = [-1/3]$.

We have looked at self-replication from logical, descriptive, recursive and biological points of view.

Key aspects of coding and reference occur with different emphasis, and we see that in each domain the intertwining of syntax and semantics takes a different form in the relation of the way that a universe divides into observer and observed.

Thank you for your attention.

APPENDIX - Quantum Formalism

Dirac [5] introduced the "bra -(c)-ket" notation $< A|B >= A^*B$ for the inner product of complex vectors $A, B \in H$. He also separated the parts of the bracket into the *bra* $< A|$ and the *ket* $|B >$. Thus

$$< A|B >=< A|\ |B >$$

Dirac can write the "ket-bra" $|A >< B| = AB^*$.

$$P = |A >< B|$$

$$P^2 = |A >< B||A >< B| = |A >< B|A >< B|$$

$$=< B|A > |A >< B| =< B|A > P.$$

Sum over Paths (Possibilities)

If $\{|C_1>, |C_2>, \cdots |C_n>\}$ is an orthonormal basis for H, and $P_i = |C_i><C_i|$, then for any vector $|A>$ we have

$$|A> = <C_1|A>|C_1> + \cdots + <C_n|A>|C_n> .$$

Hence

$$<B|A> = <C_1|A><B|C_1> + \cdots + <C_n|A><B|C_n>$$

$$= <B|C_1><C_1|A> + \cdots + <B|C_n><C_n|A>$$

$$= <B| \; [|C_1><C_1| + \cdots + |C_n><C_n|] \; |A>$$

$$= <B|\,1\,|A> .$$

$$\Sigma_{k=1}^{n} P_k = \Sigma_{k=1}^{n} |C_k >< C_k| = 1$$

In the quantum context one may wish to consider the probability of starting in state $|A>$ and ending in state $|B>$. The square of the probability for this event is equal to $| < B|A > |^2$. This can be refined if we have more knowledge. If it is known that one can go from A to C_i $(i = 1, \cdots, n)$ and from C_i to B and that the intermediate states $|C_i>$ are a complete set of orthonormal alternatives then we can assume that $< C_i|C_i >= 1$ for each i and that $\Sigma_i |C_i >< C_i| = 1$. This identity now corresponds to the fact that 1 is the sum of the probabilities of an arbitrary state being projected into one of these intermediate states.

We compare

$$E = |C >< W|$$

and

$$1 = \Sigma_k |C_k >< C_k|.$$

That the unit 1 can be written as a sum over the intermediate states is an expression of how the environment (in the sense of the space of possibilities) impinges on the quantum amplitude, just as the expression of the environment as a soup of bases ready to be paired (a classical space of possibilities) serves as a description of the biological environment. The symbol $E = |C >< W|$ indicated the availability of the bases from the environment to form the complementary pairs. The projection operators $|C_i >< C_i|$ are the possibilities for interlock of initial and final state through an intermediate possibility. In the quantum mechanics the special pairing is not of bases but of a state and a possible intermediate from a basis of states. It is through this common theme of pairing that the conceptual notation of the bras and kets lets us see a correspondence between such separate domains.

Proof of the No Cloning Theorem. In order to have a quantum process make a copy of a quantum state we need a unitary mapping $U : H \otimes H \longrightarrow H \otimes H$ where H is a complex vector space such that there is a fixed state $|X> \in H$ with the property that

$$U(|X>|A>) = |A>|A>$$

for any state $|A> \in H$. ($|A>|B>$ denotes the tensor product $|A> \otimes |B>$.) Let

$$T(|A>) = U(|X>|A>) = |A>|A> .$$

Note that T is a linear function of $|A>$. Thus we have

$$T|0> = |0>|0> = |00>,$$

$$T|1> = |1>|1> = |11>,$$

$$T(\alpha|0> + \beta|1>) = (\alpha|0> + \beta|1>)(\alpha|0> + \beta|1>).$$

But

$$T(\alpha|0> + \beta|1>) = \alpha|00> + \beta|11> .$$

Hence

$$\alpha|00> + \beta|11> = (\alpha|0> + \beta|1>)(\alpha|0> + \beta|1>)$$

$$= \alpha^2|00> + \beta^2|11> + \alpha\beta|01> + \beta\alpha|10>$$

From this it follows that $\alpha\beta = 0$. Since α and β are arbitrary complex numbers, this is a contradiction. □

The proof of the no-cloning theorem depends crucially on the linear superposition of quantum states and the linearity of quantum process. By the time we reach the molecular level and attain the possibility of copying DNA molecules we are copying in a quite different sense than the ideal quantum copy that does not exist. The DNA and its copy are each quantum states, but they are different quantum states! That we see the two DNA molecules as identical is a function of how we filter our observations of complex and entangled quantum states. Nevertheless, the identity of two DNA copies is certainly at a deeper level than the identity of the two letters "i" in the word identity. The latter is conventional and symbolic. The former is a matter of physics and biochemistry.